Energy of a Conformal Scalar Dyon Black Hole

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We obtain the energy of a conformal scalar dyon black hole (CSD) by using the energy-momentum complexes of Tolman and Møller. The total gravitational energy is given by the CSD charge in the both prescriptions.

I. INTRODUCTION

The localization of energy is a long-standing problem in the theory of general relativity. Also, it is claimed that the energy cannot be localized. We do not share this opinion.

It is possible to evaluate the energy and momentum distribution by using various energy-momentum complexes. There exist an opinion that the energy-momentum complexes are not useful to get meaningful energy distribution in a given geometry. Virbhadra and his collaborators re-opened the problem of the energy-momentum localization by using the energy-momentum complexes. The results obtained for some space-times lead to the conclusion that different energy-momentum complexes give the same energy distribution for a given space-time [1]-[12]. Recently, Virbhadra [13] used some of these results and investigated the Seifert conjecture. The author calculated the energy distribution of a dilaton dyonic black hole [14] and the energy of the Bianchi type I solution [15]. Also, we obtained the energy distribution in a static spherically symmetric nonsingular black hole space-time [16].

The purpose of this paper is to compute the energy distribution of a conformal scalar dyon black hole by using the Tolman and Møller prescriptions. We use the geometrized units ($G = 1, c = 1$) and follow the convention that the Latin indices run from 0 to 3.

II. ENERGY IN THE TOLMAN PRESCRIPTION

Virbhadra [17] gave an exact solution of Einstein–Maxwell conformal scalar field equations which is a black hole solution and is characterized by the scalar charge, electric charge and magnetic charge. This solution is given by the line element

$$ds^2 = \left(1 - \frac{Q_{CSD}}{r}\right)^2 dt^2 - \left(1 - \frac{Q_{CSD}}{r}\right)^{-2} dr^2 - \\ - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (2.1)$$

and the conformal scalar field is given by

$$\psi = \sqrt{\frac{3}{4\pi}} \left(\frac{q_s}{r - Q_{CSD}}\right) \quad (2.2)$$

with q_s the scalar charge, and where

$$Q_{CSD} = \sqrt{q_s^2 + q_e^2 + q_m^2} \quad (2.3)$$

In (2.3) q_e and q_m are the electric charge and, respectively, the magnetic charge.

The only non-zero components of the electromagnetic field tensor are

$$F_{rt} = \frac{q_e}{r^2}, \qquad F_{\theta\varphi} = q_m \sin\theta. \quad (2.4)$$

The solution given by (2.1) has an event horizon at $r = Q_{CSD}$. Also, this solution is the magnetic generalization of Bekenstein's solution [18].

Tolman's energy-momentum complex [19] is given by

$$\gamma_i{}^k = \frac{1}{8\pi} U_i{}^{kl}{}_{,l}, \quad (2.5)$$

where $\gamma_0{}^0$ and $\gamma_\alpha{}^0$ are the energy and momentum components. We have

$$U_i{}^{kl} = \sqrt{-g}\left(-g^{pk}V_{ip}{}^l + \frac{1}{2}g_i{}^k g^{pm}V_{pm}{}^l\right) \quad (2.6)$$

with

$$V_{jk}{}^i = -\Gamma_{jk}^i + \frac{1}{2}g_j{}^i\Gamma_{mk}^m + \frac{1}{2}g_k{}^i\Gamma_{mj}^m. \quad (2.7)$$

Also, the energy-momentum complex $\gamma_i{}^k$ satisfies the local conservation laws

$$\frac{\partial\gamma_i{}^k}{\partial x^k} = 0. \quad (2.8)$$

The energy and momentum in Tolman's prescription are given by

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$$P_i = \iiint \gamma_i{}^0 dx^1 dx^2 dx^3. \qquad (2.9)$$

Using the Gauss theorem we obtain

$$P_i = \frac{1}{8\pi} \iint U_i{}^{0\alpha} n_\alpha dS, \qquad (2.10)$$

where $n_\alpha = \left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$ are the components of a normal vector over an infinitesimal surface element $dS = r^2 \sin\theta d\theta d\varphi$.

We get the expression of the energy in the Tolman prescription in the case of a general spherically symmetric space-time that is described by the line element

$$ds^2 = B(r)dt^2 - A(r)dr^2 - D(r)r^2(d\theta^2 + \sin^2\theta d\varphi^2). \qquad (2.11)$$

The energy is given by

$$E(r) = \frac{r}{2} \cdot \frac{\sqrt{A(r)B(r)}\,(A(r) - D(r))}{A(r)}. \qquad (2.12)$$

After some calculations, by using (2.12) we obtain for the CSD black hole

$$E(r) = Q_{CSD}\left(1 - \frac{Q_{CSD}}{2r}\right). \qquad (2.13)$$

The total gravitational energy of a CSD black hole which is obtained for $r \to \infty$ in (13) is equal to its CSD charge.

III. ENERGY IN THE MØLLER PRESCRIPTION

Møller's energy-momentum complex [20] is given by

$$\Theta_i{}^k = \frac{1}{8\pi} \cdot \frac{\partial \chi_i{}^{kl}}{\partial x^l}, \qquad (3.1)$$

where

$$\chi_i{}^{kl} = \sqrt{-g}\left(\frac{\partial g_{in}}{\partial x^m} - \frac{\partial g_{im}}{\partial x^n}\right) g^{km} g^{ln}. \qquad (3.2)$$

The energy in the Møller prescription is given by

$$E = \iiint \Theta_0{}^0 dx^1 dx^2 dx^3 = \frac{1}{8\pi} \frac{\partial \chi_0{}^{0l}}{\partial x^l} dx^1 dx^2 dx^3. \quad (3.3)$$

The Møller energy-momentum complex is not necessary to carry out the calculation in the quasi-Cartesian coordinates, so we can calculate in the spherical coordinates.

For the line-element given by (2.1) the $\chi_0{}^{01}$ component is

$$\chi_0{}^{01} = \frac{2(r - Q_{CSD})Q_{CSD}\sin\theta}{r} \qquad (3.4)$$

Now, substituting (3.4) in (3.3) and applying the Gauss theorem we obtain the energy of the CSD black hole

$$E(r) = Q_{CSD}\left(1 - \frac{Q_{CSD}}{r}\right). \qquad (3.5)$$

In the Møller prescription the second term in the expression of the energy is twice the value obtained by using the Tolman energy-momentum complex.

It is important to note that the total gravitational energy of the CSD black hole has the same expression as in the Tolman prescription and is given by the CSD charge.

IV. DISCUSSION

The main purpose of the present paper is to show that it is possible to "solve" the problem of the localization of energy in relativity by using the energy-momentum complexes.

Bondi [21] sustained that a nonlocalizable form of energy is not admissible in relativity so its location can in principle be found. Some interesting results which have been found recently show that the several energy-momentum complexes can give the same and acceptable result for a given space-time. Also, in his recent paper Virbhadra [13] emphasized that though the energy-momentum complexes are non-tensors under general coordinate transformations, the local conservation laws with them hold in all coordinate systems. Chang, Nester and Chen [22] showed that the energy-momentum complexes are actually quasilocal and legitimate expressions for the energy-momentum.

We have calculated the energy of a CSD black hole by using the Tolman and Møller energy-momentum complexes. In the Tolman prescription the energy associated with the CSD black hole is found to be the same as it was earlier evaluated by Virbhadra in the Weinberg prescription. The Møller energy-momentum complex gives for the second term in the expression of the energy twice the value obtained by using the Tolman energy-momentum complex. The total gravitational energy of the CSD black hole obtained when $r \to \infty$ is the same in the both Tolman and Møller prescriptions and is equal to the CSD charge.

Also, we obtain the expression of the energy distribution in the case of a general static spherically symmetric space-time by using the Tolman prescription.

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